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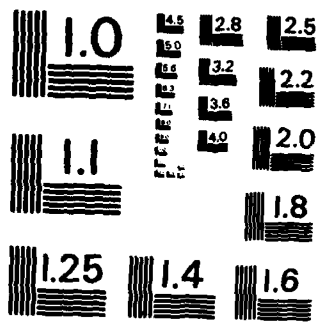

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The original proposal to AFOSR suggested that the major emphasis of this research would be on "Solving Elliptic-Parabolic Problems." Three of the topics of special interest were: (1) The extension of the basic theory (originally developed for finite-difference equations) of Professor Parter and his co-workers to those algebraic systems which arise in the finite-element approach to elliptic and parabolic problems. (2) The study of multi-grid methods. (3) A continuing experimental/analytical study of special splittings in the generalized conjugate gradient methods (CONT.)

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ITEM #20, CONT.: for elliptic and parabolic problems. This interim report documents progress to date for the first year of the grant.

AFOSR-TR- 83-0824

Annual Report

Air Force Grant AFOSR-82-0275

For Period: June 15, 1982 - June 14, 1983

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Seymour V. Parter

Seymour V. Parter
Principal Investigator

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Goals of the Grant

The original proposal to AFOSR suggested that the major emphasis of this research would be on "Solving Elliptic-Parabolic Problems". Three ~~of the~~ topics of special interest ~~were:~~ follow.

(1) The extension of the basic theory (originally developed for finite-difference equations) of Professor Parter and his co-workers to those algebraic systems which arise in the finite-element approach to elliptic and parabolic problems.

(2) The study of multi-grid methods.

(3) A continuing experimental/analytical study of special splittings in the generalized conjugate gradient methods for elliptic and parabolic problems.

2. Progress to Date

In connection with the first goal; i.e, extending the basic theory to finite-element problems, one report has been written: Block Iterative Methods for Elliptic Finite-Element Equations. This 69 page report is authored by Seymour V. Parter and Michael Steuerwalt (of the Los Alamos National Laboratory). This work has been submitted for publication in the SIAM Journal on Numerical Analysis. We have every reason to believe it will be accepted for Journal publication. This report contains a general theory - including a new convergence proof for a large class of iterative methods. This proof does not require that the matrices involved be symmetric nor does it require that the matrices be of positive-type or inverse monotone. Of course, the fact that these matrices arise from the discretization of elliptic problems is essential. The second part of this report deals with the application of this theory to a special class of problems.

Let $\Omega := \{(x,y);\ 0 < x,y < 1\}$ be the unit square. Let

$$L := \sum a_{ij} \frac{\partial^2}{\partial x_i \partial x_j} + \sum a_i \frac{\partial}{\partial x_i} + c\,, \quad i,j = 1,2$$

where

$$c(x,y) \leq 0\,,$$

be a uniformly elliptic (2nd order) operator defined on Ω. Consider the Dirichlet problem

$$Lu = f \,, \quad \Omega$$
$$u = 0 \,, \quad \partial\Omega \,.$$

Consider also a finite-element approach for the numerical solution of this problem. Let the finite-element method be based on the finite-dimensional subspaces given by the tensor products of Hermite Cubic Splines. Let us now solve these equations by k-line (horizontal lines) iterative methods. The results are: Let Λ_0 be the smallest eigenvalue of the problem

$$L\phi + \Lambda c\phi = 0 \,:\, \Omega$$
$$\phi = 0 \,:\, \partial\Omega \,.$$

Then

A) for the k-line Jacobi iterative method

$$\rho \approx 1 - \frac{5}{12} k \Lambda_0 h^2 \,,$$

B) for the k-line SOR iterative method with the optimal ω

$$\rho_b \approx 1 - 2\left[\frac{5k}{6}\Lambda_0\right]^{\frac{1}{2}} h \,.$$

The method is quite powerful as a "comparer" even when complete results are not available. For example, in the case of the "point" Gauss-Seidel method we can not give a complete answer. However, we have the following result. Let

$$q(x,y) := \frac{156}{175}[a_{11}(x,y) + a_{22}(x,y)]$$

and let Λ_0 now be the smallest eigenvalue of the problem

$$L\phi + \Lambda_0\phi = 0 : \Omega ,$$
$$\phi = 0 : \partial\Omega .$$

Let ρ denote the spectral radius of the point Gauss Seidel method. Then

$$\rho \geq 1 - \Lambda_0 h^2 + 0(h^2) .$$

With this report we feel that this topic has been essentially resolved. There are several computational projects being organized in connection with this work. These projects will highlight some of the salient points as well as investigate certain delicate questions left unresolved. Professor Parter will report on this work as part of his "Retiring Presidential Address" at the SIAM 1983 Fall meeting, November 7-9, 1983 in Norfolk, VA. Also, he will present a report on the computational results and related topics at the Sixth International Symposium on Computing Methods in Engineering and Applied Science to be held December 12-16, 1983 in Paris, France.

In connection with the study of multi-grid methods a first basic project has begun. There is preliminary theoretical work on two topics:

(a) 1-dimensional problems.

(b) MGR[0] methods for problems with variable coefficients.

In both of these topics the theoretical work is based on a thorough analysis of "two-grid" methods. Computational experiments are now being planned and organized.

Finally, on the topic of Conjugate Gradient methods, a small experimental program has been outlined. This program is based on certain questions which arise in the analysis of "Block Iterative Methods for Elliptic Finite-Element Equations". In simple terms, the analysis of the full problem was reduced to the analysis of a related, but simpler, finite-difference system. The conjugate gradient program will study these two problems and try to relate the "rates of convergence" for the conjugate gradient methods in these cases.

SUMMARY

Reports Completed:

Block Iterative Methods for Elliptic Finite-Element Equations (with M. Steuerwalt)

Work in Progress (begun during the period 6/15/82-6/14/83)

Remarks on Multi-grid for 1-dimensional Problems

Remarks on MGR Multigrid Methods

Computational Studies on Point S.O.R. for Elliptic Finite-Element Equations

Computational Studies on k-line Iterative Methods for Elliptic Finite-element Equations

Computational Studies on k-line Conjugate Gradient Methods for Elliptic Finite-element Equations

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- 8